Extending Special Relativity to Superluminal Motion

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Abstract

Experiments done with single photon in the early 1990's produced a surprising result: that single photon pass through a photon tunnel barrier with a group velocity faster than the vacuum speed of light. Recently, a series of experiments revealed that electromagnetic wave was able to travel at a group velocity faster than c. These phenomena have been observed in dispersive media. We think all particles can be divided into three kinds in nature: The first kind of particle is its velocity in the range of $0 \leq v < c$, e.g. electron, atom and so on. The second kind of particle is its velocity in the range of $0 \leq v < c_m$, e.g. photon. The third kind of particle is its velocity in the range of $c \leq v < c_m$ (c_m is the maximum velocity in nature), e.g. tachyon. The first kind of particle is described by the special relativity. In this paper, we give some new kinematic and dynamic equations to describe the second and third kinds particles.

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1. Introduction

About one hundred years ago, Einstein laid the foundation for a revolution in the conception of space and time, matter and energy. Later, special theory of relativity was accepted by mainstream physicists. It is based on two postulates by Einstein [1]:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames. In other words, we can say that the equation expressing the laws of nature are invariant with respect to transformations of coordinates and time from one inertial reference frame to another.

2. The Universal Speed of Light: The speed of light in vacuum is the same for all inertia observers, regardless of the motion of the source, the observer, or any assumed medium of propagation.

The invariant principle of the speed of light is right in all inertial reference frames in which their relative velocity u is less than the speed of light c. Since the light velocity has no relation with the movement of light source, which has been proved by experiment [2], we can treat a moving light source as an inertial reference frame, and we can obtain the result that the speed of light has nothing to do with the moving speed of the inertial reference frame, i.e., the speed of light is the same in all inertial reference frames. It derives that light has no interaction with light source, and the light has no inertia. So, the rest mass of light tend to zero. Recently, a series of experiments revealed that electromagnetic wave was able to travel at a group velocity faster than c. These phenomena have been observed in dispersive media [3, 4], in electronic circuits [5], and in evanescent wave cases [6]. In fact, over the last decade, the discussion of the tunnelling time problem has experienced a new stimulus by the results of analogous experiments with evanescent electromagnetic wave packets [7], and the superluminal effects of evanescent waves have been revealed in photon tunnelling experiments in both the optical domain and the microwave range [6]. In nature, there is superluminal phenomena, and the relative velocity u between two inertial reference frames can be larger than c or equal to c. Otherwise, the superluminal phenomena can also appear in the progress of light propagation. For example, when a beam of light moves at the same direction, the relative velocity u between all photons is equal to zero, it is not the speed of light c. So, the postulation about the invariant principle of the speed of light shouldn't be correct when the relative velocity of two inertial

reference frames is equal to the speed of light (we regard the light as an inertial reference frames). When two beams of light move at the opposite direction, their relative velocity u exceeds the speed of light c, the result that the speed of light c is the maximum speed shouldn't be correct. So, when two inertial reference frames relative velocity u is larger than the speed of light or equal to the speed of light, Einstein's invariant principle of the speed of light should be modified. In nature, particles can be divided into three kinds. The first kind of particle's velocity is in the range of $0 \leq v < c$, e.g. electron, atom and so on. The second kind of particle's velocity is in the range of $0 \leq v < c_m$, e.g. photon. The third kind of particle's velocity is in the range of $c \leq v < c_m$ (c_m is the maximum velocity in nature), e.g. tachyon. The first kind of particle is described by the special relativity. The second and third kinds of particle will be researched in this paper.

2. The space-time transformation and mass-energy relation for the first kind of particle ($0 \leq v < c$)

The first kind of particle's velocity is in the range of $0 \leq u < c$, e.g. electron, atom and so on, and they can be described by the special relativity. In 1905, Einstein gave the space-time transformation and mass-energy relation which are based on his two postulates. The space-time transformation is

$$
\begin{aligned}
x &= \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c^2}}} \\
y &= y' \\
z &= z' \\
t &= \frac{t' + \frac{u}{c^2}x'}{\sqrt{1 - \frac{u^2}{c^2}}},
\end{aligned}
\tag{1}
$$

where x, y, z, t are space-time coordinates in $\sum$ frame, x', y', z', t' are space-time coordinates in $\sum'$ frame, c is the speed of light, and u is the relative velocity between $\sum$ and $\sum'$ frame, which move along x and x' axes. The velocity transformation is

$$
u_x = \frac{u'_x + u}{1 + \frac{uu'_x}{c^2}}
$$

$$u_y = \frac{u_y'\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{uu_x'}{c^2}}$$

$$u_z = \frac{u_z'\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{uu_x'}{c^2}}, \tag{2}$$

where u_x, u_y and u_z are a particle velocity projection in $\sum$ frame, u_x', u_y' and u_z' are the particle velocity projection in $\sum'$ frame. The relation between a particle mass m and its velocity v is

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{3}$$

with m_0 and m being the particle rest mass and relativistic mass respectively. The relation between a particle relativistic energy E and its relativistic mass m is

$$E = mc^2, \tag{4}$$

and the relation of particle energy E with its momentum p is

$$E^2 = m_0^2 c^4 + p^2 c^2. \tag{5}$$

3. The space-time transformation and mass-energy relation for the second kind of particle ($0 \leq v < c_m$)

The second kind of particle's velocity is in the range of $0 \leq v < c_m$ ($c < c_m$), e.g. photon. Recently, a series of experiments revealed that electromagnetic wave was able to travel at a group velocity faster than c. These phenomena have been observed in dispersive media [3, 4], in electronic circuits [5], and in evanescent wave cases [6]. It is about 40 years before, that O.M.P. Bilaniuk, V.K. Deshpande and E.S.G. Sudarshan have studied the space-time relation for superluminal reference frames within the framework of special relativity [8, 9]. They assumed that the space-time and velocity transformation of special relativity are suitable for superluminal reference frames. They obtained the new results that the proper length L_0 and proper time T_0 must be imaginary so that the measured quantities, such as length L and time T, are real. In this paper, we extend the framework of special relativity. We think there is superluminal particle, but the particle velocity can not be infinity. So, we can assume that there is a limit velocity in nature, which is called the maximum velocity c_m. All particle movement velocities can not exceed the maximum

velocity c_m in arbitrary inertial reference frame. In the velocity range of $0 \leq v < c_m$, we give two postulates as follows:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames.

2. The Universal of Maximum Velocity: There is a maximum velocity c_m in nature, and the c_m is invariant in all inertial reference frames.

From the two postulates, we can obtain the space-time transformation and velocity transformation for the second kind of particle ($0 \leq v < c_m$). When we replace c with c_m, we can obtain the new transformation from the Lorentz transformation, they are

$$
\begin{aligned}
x &= \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c_m^2}}} \\
y &= y' \\
z &= z' \\
t &= \frac{t' + \frac{u}{c_m^2}x'}{\sqrt{1 - \frac{u^2}{c_m^2}}},
\end{aligned}
\tag{6}
$$

where x, y, z, t are space-time coordinates in $\sum$ frame, x', y', z', t' are space-time coordinates in $\sum'$ frame, c is the speed of light, and u is the relative velocity between $\sum$ and $\sum'$ frame, which move along x and x' axes. The velocity transformation is

$$
\begin{aligned}
u_x &= \frac{u'_x + u}{1 + \frac{uu'_x}{c_m^2}} \\
u_y &= \frac{u'_y\sqrt{1 - \frac{u^2}{c_m^2}}}{1 + \frac{uu'_x}{c_m^2}} \\
u_z &= \frac{u'_z\sqrt{1 - \frac{u^2}{c_m^2}}}{1 + \frac{uu'_x}{c_m^2}},
\end{aligned}
\tag{7}
$$

where u_x, u_y, u_z and $v = \sqrt{u_x^2 + u_y^2 + u_z^2}$ ($0 \leq v < c_m$) are a particle velocity component and velocity amplitude in $\sum$ frame, u'_x, u'_y, u'_z and $v' = \sqrt{u'^2_x + u'^2_y + u'^2_z}$ ($0 \leq v' < c_m$) are the particle velocity component projection and velocity amplitude in $\sum'$ frame. Now, We can discuss the problem of the speed of light. For two inertial reference frames $\sum$ and $\sum'$, the $\sum'$ frame is a rest frame for light, i.e., the two reference frames relative velocity v

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is equal to c. At the time $t = 0$, a beam of light is emitted from the origin O. From Eq. (7), we have

$$u_x = c, \tag{8}$$

then

$$u'_x = 0, \tag{9}$$

and

$$u_x = -c, \tag{10}$$

and then

$$u'_x = \frac{-c - c}{1 + \frac{c^2}{c_m^2}} = -2\frac{cc_m^2}{c^2 + c_m^2} > -2c. \tag{11}$$

It shows that the invariant principle of light velocity is violated in the inertial reference of light velocity movement. The relation between a particle mass m and its movement velocity v is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_m^2}}}, \tag{12}$$

with m_0 and m being the particle rest mass and relativistic mass respectively. The relation between a particle relativistic energy E and its relativistic mass m is

$$E = mc_m^2, \tag{13}$$

and the relation between particle energy E and its momentum p is

$$E^2 = m_0^2 c_m^4 + p^2 c_m^2. \tag{14}$$

In the following, we study the nature of photon. From Eqs. (12) and (13), we have

$$m_\nu c_m^2 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_m^2}}} c_m^2 = h\nu, \tag{15}$$

where m_ν is photon mass, ν is photon frequency when its velocity is v $(0 \leq v < c_m)$. When $v = 0$, we have

$$m_0 = \frac{h}{c_m^2} \nu_0, \tag{16}$$

where m_0 and ν_0 are photon rest mass and rest frequency respectively. From Eqs. (15) and (16), we have

$$\nu = \frac{m_0 c_m^2}{h\sqrt{1 - \frac{v^2}{c_m^2}}} = \frac{\nu_0}{\sqrt{1 - \frac{v^2}{c_m^2}}}, \tag{17}$$

The Eq. (17) gives the relation of a photon's frequency with its velocity, and the relation of a photon's movement frequency ν with rest frequency ν_0. From the relation

$$\nu\lambda = v, \tag{18}$$

i.e.

$$\frac{m_0 c_m^2}{h} \frac{1}{\sqrt{1 - \frac{v^2}{c_m^2}}} \lambda = v, \tag{19}$$

then

$$\lambda = \frac{hv}{m_0 c_m^2} \sqrt{1 - \frac{v^2}{c_m^2}}, \tag{20}$$

The Eq. (20) gives the relation of a photon's wavelength with its velocity,. From Eq. (15) and (17), we can obtain the relation between photon mass m and it velocity v ($0 \leq v < c_m$)

$$m_\nu = \frac{h\nu}{c_m^2} = \frac{h\nu_0}{c_m^2 \sqrt{1 - \frac{v^2}{c_m{}^2}}}. \tag{21}$$

From Eq. (21), we find the photon mass m_ν is directly proportional to ν, and we can calculate the maximum velocity c_m if we can measure the photon mass m_ν when its frequency is ν. All photon possess a finite mass and their physical implications have been discussed by many theories and experiments [10, 11, 12]. In Ref. [11, 12], the experiment were made by laser, and it determined the lower limit of the photon mass $10^{-6}eV < m_\nu < 10^{-4}eV$, we know the laser frequency is in the range of $8.9 \times 10^{13}Hz \sim 9.23 \times 10^{14}Hz$. From Eq. (21), we can estimate the maximum velocity c_m. It is in the rang of: $\sqrt{\frac{6.626 \times 10^{-34} \times 8.9 \times 10^{13}}{10^{-4} \times 1.6 \times 10^{-19}}}c \leq c_m = \sqrt{\frac{h\nu}{m_\nu}} \leq \sqrt{\frac{6.626 \times 10^{-34} \times 9.23 \times 10^{14}}{10^{-6} \times 1.6 \times 10^{-19}}}c$, i.e., $60c \leq c_m \leq 2000c$. In Ref. [13], the experiment measured the superluminal velocity is $310c$. In Refs. [14-15], the experiments measured signal velocity were $4.7c$ for the microwave and $1.7c$ for single photon.

4. The space-time transformation and mass-velocity relation for the third kind of particle ($c \leq v < c_m$)

The third kind of particle's velocity is in the range of $c \leq v < c_m$ ($c < c_m$), e.g. tachyon. Tachyon arises naturally in superstring theory, and the detailed analysis from algebraic as well as space-time diagram viewpoints is given in [16]. There are two well-known vacuum in open bosonic string field theory (OSFT) [17]. One is the unstable (perturbative)

vacuum and the other is the tachyon (nonperturbative) vacuum. M. Schnabl obtained the analytic solution for the tachyon vacuum [18]. After the Schnabls work, there has been remarkable progress in understanding of OSFT [19]. Especially, many works have been devoted to the construction of analytic solutions in bosonic string [20], and superstring field theories[21, 22].

In the following, we will give the relation of space-time in two inertial reference frames $\sum$ and $\sum'$ for the third kind of particle $(c \leq v < c_m)$. We think the tachyon travels faster than light, but its velocity can not be infinity. So, we can assume there is a limit velocity c_m in nature. For the tachyon $(c \leq v < c_m)$, we also give two postulates as follows:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames.

2. The Universal of Maximum Velocity: There is a maximum velocity c_m in nature, and the c_m is invariant in all inertial reference frames.

From the two postulates, we can obtain the space-time transformation and velocity transformation for the third kind of particle $(c \leq v < c_m)$. When we replace c with c_m, we can obtain the new transformation from the Lorentz transformation, they are

$$
\begin{aligned}
x &= \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c_m^2}}} \\
y &= y' \\
z &= z' \\
t &= \frac{t' + \frac{u}{c_m^2}x'}{\sqrt{1 - \frac{u^2}{c_m^2}}},
\end{aligned}
\tag{22}
$$

where x, y, z, t are space-time coordinates in $\sum$ frame, x', y', z', t' are space-time coordinates in $\sum'$ frame, c is the speed of light, and u is the relative velocity between $\sum$ and $\sum'$ frame, which move along x and x' axes. The velocity transformation is

$$
\begin{aligned}
u_x &= \frac{u_x' + u}{1 + \frac{uu_x'}{c_m^2}} \\
u_y &= \frac{u_y'\sqrt{1 - \frac{u^2}{c_m^2}}}{1 + \frac{uu_x'}{c_m^2}} \\
u_z &= \frac{u_z'\sqrt{1 - \frac{u^2}{c_m^2}}}{1 + \frac{uu_x'}{c_m^2}},
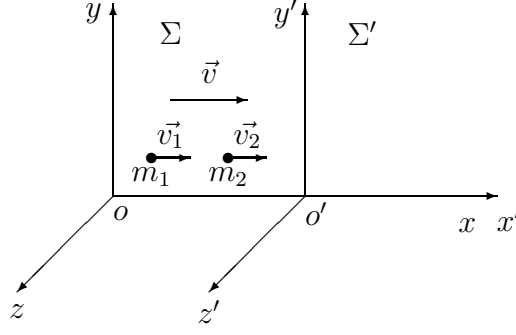\end{aligned}
\tag{23}
$$

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Figure 1: The Σ is the laboratory system, Σ' is the mass-center system for two particles m_1 and m_2, and the two inertial frames relative velocity is v.

where u_x, u_y, u_z and $v = \sqrt{u_x^2 + u_y^2 + u_z^2}$ $(c \leq v < c_m)$ are a particle velocity component and velocity in Σ frame, u'_x, u'_y, u'_z and $v' = \sqrt{u'^2_x + u'^2_y + u'^2_z}$ $(c \leq v' < c_m)$ are the particle velocity component and velocity in Σ' frame. In the following, we will give the new relation of particle mass m with its velocity v. We can consider the collision between two identical particle. It is shown in Figure 1.

The Σ is the laboratory system, and Σ' is the mass-center system of two particles m_1 and m_2. In Σ system, the velocity of two particles m_1 and m_2 are $\vec{v_1}$ and $\vec{v_2}$ $(c \leq v_2 < v_1 < c_m)$, which are along with $x(x')$ axis, and they are v' and $-v'$ in Σ' system. After collision, the two particles velocities are all v $(c \leq v < c_m)$ in Σ system. The momentum was conserved in this process:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v. \tag{24}$$

According to equation (23),

$$v_1 = \frac{v' + v}{1 + \frac{vv'}{c_m^2}}$$

$$v_2 = \frac{-v' + v}{1 - \frac{vv'}{c_m^2}}. \tag{25}$$

From equations (24) and (25), we get

$$m_1 \left(1 - \frac{vv'}{c_m^2}\right) = m_2 \left(1 + \frac{vv'}{c_m^2}\right), \tag{26}$$

from equation (23), we can obtain

$$1 - \frac{u'_x u}{c_m^2} = \frac{\sqrt{1 - \frac{v^2}{c_m^2}} \sqrt{1 - \frac{u^2}{c_m^2}}}{\sqrt{1 - \frac{v'^2}{c_m^2}}}, \tag{27}$$

where $v = \sqrt{u_x^2 + u_y^2 + u_z^2}$ and $v' = \sqrt{u'^2_x + u'^2_y + u'^2_z}$. For the particle m_1, the equation (27) becomes

$$1 + \frac{v'v}{c_m^2} = \frac{\sqrt{1 - \frac{v'^2}{c_m^2}}\sqrt{1 - \frac{v^2}{c_m^2}}}{\sqrt{1 - \frac{v_1^2}{c_m^2}}}. \tag{28}$$

For the particle m_2, the equation (27) becomes

$$1 - \frac{v'v}{c_m^2} = \frac{\sqrt{1 - \frac{v'^2}{c_m^2}}\sqrt{1 - \frac{v^2}{c_m^2}}}{\sqrt{1 - \frac{v_2^2}{c_m^2}}}. \tag{29}$$

By substituting equations (28) and (29) into (26), we get

$$m(v_1)\sqrt{1 - \frac{v_1^2}{c_m^2}} = m(v_2)\sqrt{1 - \frac{v_2^2}{c_m^2}} = m(c)\sqrt{1 - \frac{c^2}{c_m^2}} = constant, \tag{30}$$

where $m(c)$ is the particle mass when its velocity is equal to c.

For velocity v $(c \leq v < c_m)$, we have

$$m(v)\sqrt{1 - \frac{v^2}{c_m^2}} = m(c)\sqrt{1 - \frac{c^2}{c_m^2}}, \tag{31}$$

and hence

$$m(v) = m_c\sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}}. \tag{32}$$

with $m_c = m(c)$. The equation (32) is the relation between tachyon mass m and its velocity v $(c \leq v < c_m)$.

5. The relation of energy with mass for the second and third kinds particles

For the first kind particle $(0 \leq v < c)$, the 4-vector is defined

$$x_\mu = (x_1, x_2, x_3, x_4) = (x, y, z, ict). \tag{33}$$

The invariant interval ds^2 is given by the equation

$$ds^2 = -dx_\mu dx_\mu = c^2 dt^2 - (dx)^2 - (dy)^2 - (dz)^2 = c^2 d\tau^2, \tag{34}$$

we get

$$d\tau = \frac{1}{c}ds, \tag{35}$$

where $d\tau$ is the proper time, the 4-velocity can be defined by

$$U_\mu = \frac{dx_\mu}{d\tau} = \frac{dx_\mu}{dt}\frac{dt}{d\tau} = \gamma_\mu(\vec{v}, ic), \tag{36}$$

where $\gamma_\mu = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $\vec{v} = \frac{d\vec{x}}{dt}$ and $\frac{dt}{d\tau} = \gamma_\mu$. We can define the 4-momentum as

$$p_\mu = m_0 U_\mu = (\vec{p}, ip_4), \tag{37}$$

with $\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$, $p_4 = \frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}}$. We can define the energy of a particle as

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{38}$$

then

$$p_\mu = (\vec{p}, \frac{i}{c}E), \tag{39}$$

the invariant quantity constructed from this 4-vector is

$$p_\mu p_\mu = p^2 - \frac{E^2}{c^2} = -m_0^2 c^2, \tag{40}$$

i.e.,

$$E^2 - p^2 c^2 = m_0^2 c^4. \tag{41}$$

Eq. (41) is the relation among a particle mass m, momentum $\vec{p}$ and energy E.

For the second kind particle $(0 \leq v < c_m)$, when c is replaced with c_m, we can easily obtain the relation between energy and mass, and easily obtain the relation among a particle mass m, momentum $\vec{p}$ and energy E from Eqs. (33)-(41).

$$E = \frac{m_0 c_m^2}{\sqrt{1-\frac{v^2}{c_m^2}}}, \tag{42}$$

and

$$E^2 - p^2 c^2 = m_0^2 c_m^4. \tag{43}$$

For the third kind particle $(c \leq v < c_m)$, when $c(m_0)$ is replaced with $c_m(m_c)$ we can also obtain the relation between energy and mass, and easily obtain the relation among a particle mass m, momentum $\vec{p}$ and energy E from Eqs. (33)-(41).

$$E = \frac{m_c c_m^2}{\sqrt{1-\frac{v^2}{c_m^2}}}, \tag{44}$$

and

$$E^2 - p^2 c_m^2 = m_c^2 c_m^4. \tag{45}$$

By substituting Eq. (32) into (44), we can obtain the relation of mass-energy for the tachyon $(c \le v < c_m)$.

$$E = \frac{m(v)}{\sqrt{c_m^2 - c^2}} c_m^3. \tag{46}$$

6. The relativistic dynamics for the second and third kinds particles

For the first kind particle $(0 \le v < c)$, we know the 4-force is defined as:

$$K_\mu = \frac{dp_\mu}{d\tau}. \tag{47}$$

From equation (39), we have

$$K_\mu = (\vec{K}, iK_4), \tag{48}$$

the "ordinary" force $\vec{K}$ is

$$\vec{K} = \frac{d\vec{p}}{dt} \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{d\vec{p}}{dt}, \tag{49}$$

while the fourth component

$$\begin{aligned} K_4 &= \frac{dp_4}{d\tau} = \frac{1}{c} \frac{dE}{d\tau} \\ &= \frac{1}{c} \vec{v} \cdot \vec{K}, \end{aligned} \tag{50}$$

and so

$$K_\mu = (\vec{K}, \frac{i}{c} \vec{v} \cdot \vec{K}), \tag{51}$$

the covariant equation for a particle are

$$\vec{K} = \frac{d\vec{p}}{d\tau}, \tag{52}$$

$$\vec{K} \cdot \vec{v} = \frac{dE}{d\tau}. \tag{53}$$

$$\sqrt{1 - \frac{v^2}{c^2}} \vec{K} \cdot \vec{v} = \frac{dE}{dt}, \tag{54}$$

we define force $\vec{F}$ as

$$\vec{F} = \sqrt{1 - \frac{v^2}{c^2}}\vec{K}. \tag{55}$$

From Eqs. (52)-(54), we have the relativistic dynamics equations for a particle are

$$\vec{F} = \frac{d\vec{p}}{dt}, \tag{56}$$

$$\vec{K} \cdot \vec{v} = \frac{dE}{dt}. \tag{57}$$

For the second kind particle ($0 \le v < c_m$), the relativistic dynamics equations are Eqs. (55) and (56), but some physical quantities should be modified as follows:

The 4-momentum and 4-force are

$$p_\mu = m_0 U_\mu = (\vec{p}, \frac{i}{c_m}E), \tag{58}$$

and

$$K_\mu = (\vec{K}, iK_4), \tag{59}$$

with $\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c_m^2}}}$, $p_4 = \frac{m_0 c_m}{\sqrt{1-\frac{v^2}{c_m^2}}}$, $\vec{K} = \frac{d\vec{p}}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c_m^2}}}$ and $K_4 = \frac{1}{c_m}\vec{v} \cdot \vec{K}$.

For the third kind particle ($c \le v < c_m$), the relativistic dynamics equations are also Eqs. (55) and (56), and some physical quantities also should be modified as follows:

The 4-momentum and 4-force are

$$p_\mu = m_c U_\mu = (\vec{p}, ip_4), \tag{60}$$

and

$$K_\mu = (\vec{K}, iK_4), \tag{61}$$

with $\vec{p} = \frac{m_c\vec{v}}{\sqrt{1-\frac{v^2}{c_m^2}}}$, $p_4 = \frac{m_c c_m}{\sqrt{1-\frac{v^2}{c_m^2}}}$, $\vec{K} = \frac{d\vec{p}}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c_m^2}}}$ and $K_4 = \frac{1}{c_m}\vec{v} \cdot \vec{K}$.

7. The quantum wave equation for the second and third kinds particles

For the first kind particle ($0 \le v < c$), we express E and $\vec{p}$ as operators:

$$E \to i\hbar\frac{\partial}{\partial t}$$
$$\vec{p} \to -i\hbar\nabla, \tag{62}$$

we can obtain the quantum wave equation of spin 0 particle from Eq. (41)

$$[\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \frac{m_0^2 c^4}{\hbar^2}]\Psi(\vec{r}, t) = 0, \tag{63}$$

and we can obtain the quantum wave equation of spin $\frac{1}{2}$ particle

$$i\hbar \frac{\partial}{\partial t}\Psi = [-i\hbar c\vec{\alpha} \cdot \vec{\nabla} + m_0 c^2 \beta]\Psi, \tag{64}$$

where α and β are matrixes

$$\alpha = \left(\begin{array}{cc} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{array} \right),$$

and

$$\beta = \left(\begin{array}{cc} I & 0 \\ 0 & -I \end{array} \right),$$

where $\vec{\sigma}$ are Pauli matrixes, and I is unit matrix of 2×2.

For the second kind particle ($0 \le v < c_m$), we can obtain the quantum wave equation of spin 0 particle from Eq. (43)

$$[\frac{\partial^2}{\partial t^2} - c_m^2 \nabla^2 + \frac{m_0^2 c_m^4}{\hbar^2}]\Psi(\vec{r}, t) = 0, \tag{65}$$

and we can obtain the quantum wave equation of spin $\frac{1}{2}$ particle

$$i\hbar \frac{\partial}{\partial t}\Psi = [-i\hbar c_m \vec{\alpha} \cdot \vec{\nabla} + m_0 c_m^2 \beta]\Psi. \tag{66}$$

For the third kind particle ($c \le v < c_m$), we can obtain the quantum wave equation of spin 0 particle from Eq. (45)

$$[\frac{\partial^2}{\partial t^2} - c_m^2 \nabla^2 + \frac{m_c^2 c_m^4}{\hbar^2}]\Psi(\vec{r}, t) = 0, \tag{67}$$

and we can obtain the quantum wave equation of spin $\frac{1}{2}$ particle

$$i\hbar \frac{\partial}{\partial t}\Psi = [-i\hbar c_m \vec{\alpha} \cdot \vec{\nabla} + m_c c_m^2 \beta]\Psi. \tag{68}$$

8. Conclusion

In nature, we think particles can be divided into three kinds. The first kind of particle is its velocity is in the range of $0 \le v < c$, e.g. electron, atom, macroscopical matter and so on. The second kind of particle is its velocity in the range of $0 \le v < c_m$, e.g. photon.

The third kind of particle is its velocity in the range of $c \leq v < c_m$ (c_m is the maximum velocity in nature), e.g. tachyon. The first kind of particle is described by the special relativity. In this paper, we give the new space-time transformation, the relation between mass and energy, the classic dynamics equation and the quantum wave equations for the second and third kinds particles.

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